OCR
RECOGNISING ACHIEVEMENT

Thursday 13 June 2013 – Morning

A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4723/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Find

(i) $\int (4-3x)^7 \mathrm{d}x$,

(ii) $\int (4-3x)^{-1} \mathrm{d}x$.

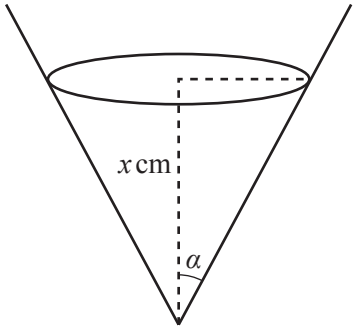
[5]

2 Using an appropriate identity in each case, find the possible values of

(i) $\sin\alpha$ given that $4\cos 2\alpha = \sin^2\alpha$, **[3]**

(ii) $\sec\beta$ given that $2\tan^2\beta = 3 + 9\sec\beta$. **[4]**

3



The diagram shows a container in the form of a right circular cone. The angle between the axis and the slant height is α, where $\alpha = \tan^{-1}(\frac{1}{2})$. Initially the container is empty, and then liquid is added at the rate of $14\,\mathrm{cm}^3$ per minute. The depth of liquid in the container at time t minutes is x cm.

(i) Show that the volume, $V\,\mathrm{cm}^3$, of liquid in the container when the depth is x cm is given by

$$V = \tfrac{1}{12}\pi x^3.$$

[The volume of a cone is $\frac{1}{3}\pi r^2 h$.] **[2]**

(ii) Find the rate at which the depth of the liquid is increasing at the instant when the depth is 8 cm. Give your answer in cm per minute correct to 2 decimal places. **[3]**

4 Find the exact value of the gradient of the curve

$$y = \sqrt{4x-7} + \frac{4x}{2x+1}$$

at the point for which $x = 4$. **[6]**

5 **(i)** Give full details of a sequence of two transformations needed to transform the graph of $y = |x|$ to the graph of $y = |2(x+3)|$. **[3]**

(ii) Solve the inequality $|x| > |2(x+3)|$, showing all your working. **[5]**

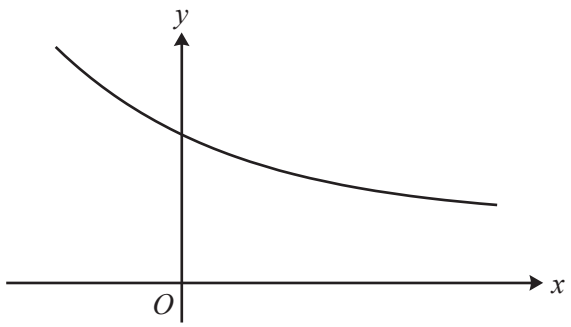
6 The value of $\int_0^8 \ln(3 + x^2)\,\mathrm{d}x$ obtained by using Simpson's rule with four strips is denoted by A.

(i) Find the value of A correct to 3 significant figures. **[4]**

(ii) Explain why an approximate value of $\int_0^8 \ln(9 + 6x^2 + x^4)\,\mathrm{d}x$ is $2A$. **[2]**

(iii) Explain why an approximate value of $\int_0^8 \ln(3\mathrm{e} + \mathrm{e}x^2)\,\mathrm{d}x$ is $A + 8$. **[2]**

7



The diagram shows the curve $y = \mathrm{f}(x)$, where f is the function defined for all real values of x by

$$\mathrm{f}(x) = 3 + 4\mathrm{e}^{-x}.$$

(i) State the range of f. **[1]**

(ii) Find an expression for $\mathrm{f}^{-1}(x)$, and state the domain and range of f^{-1}. **[4]**

(iii) The straight line $y = x$ meets the curve $y = \mathrm{f}(x)$ at the point P. By using an iterative process based on the equation $x = \mathrm{f}(x)$, with a starting value of 3, find the coordinates of the point P. Show all your working and give each coordinate correct to 3 decimal places. **[4]**

(iv) How is the point P related to the curves $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$? **[1]**

8 **(i)** Express $4\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

(ii) Hence

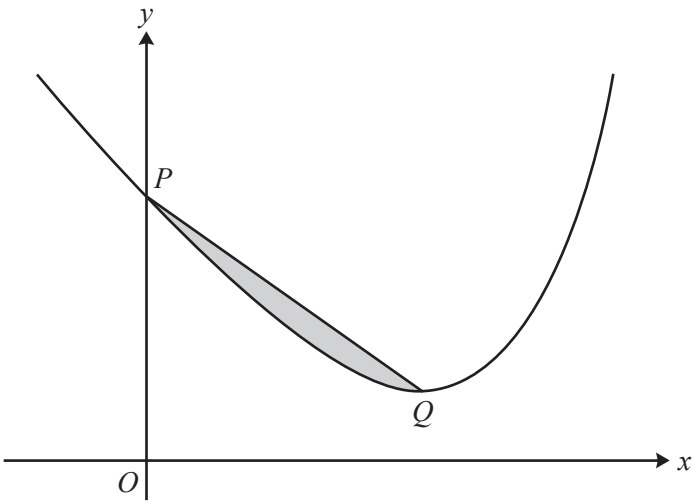
(a) solve the equation $4\cos\theta - 2\sin\theta = 3$ for $0° < \theta < 360°$, **[4]**

(b) determine the greatest and least values of

$$25 - (4\cos\theta - 2\sin\theta)^2$$

as θ varies, and, in each case, find the smallest positive value of θ for which that value occurs. **[5]**

9



The diagram shows the curve

$$y = e^{2x} - 18x + 15.$$

The curve crosses the y-axis at P and the minimum point is Q. The shaded region is bounded by the curve and the line PQ.

(i) Show that the x-coordinate of Q is $\ln 3$. **[3]**

(ii) Find the exact area of the shaded region. **[8]**

Question			Answer	Marks	Guidance
1	(i)		Obtain integral of form $k(4-3x)^8$	M1	any non-zero constant k; using substitution to obtain ku^8 earns M1
			Obtain $-\frac{1}{24}(4-3x)^8$	A1	or unsimplified equiv; must be in terms of x
1	(ii)		Obtain integral of form $k\ln(4-3x)$	M1	any non-zero constant k; allow M1 if brackets missing; using substitution to obtain $k\ln u$ earns M1; $\log(4-3x)$ with base e not specified is M1A0
			Obtain $-\frac{1}{3}\ln(4-3x)$	A1	now with either brackets or modulus signs; must be in terms of x; note that $-\frac{1}{3}\ln(x-\frac{4}{3})$ and $-\frac{1}{3}\ln(\frac{4}{3}-x)$ are correct alternatives
			Include + c or + k at least once	B1	anywhere in solution to question 1; this mark available even if no other marks earned
				[5]	
2	(i)		Use $2\cos^2\alpha-1$ or $\cos^2\alpha-\sin^2\alpha$ or $1-2\sin^2\alpha$	B1	
			Obtain equation in which $\sin^2\alpha$ appears once	M1	condoning sign slips or arithmetic slips; for solution which gives equation involving $\tan^2\alpha$, M1 is not earned until valid method for reaching $\sin\alpha$ is used; attempt involving $4(1-s^2)=s^2$ is M0
			Obtain $\pm\frac{2}{3}$	A1	both values needed; ±0.667 is A0; $\pm\sqrt{\frac{4}{9}}$ is A0; ignore subsequent work to find angle(s)
				[3]	
2	(ii)		Either Attempt use of identity	M1	of form $\tan^2\beta=\pm\sec^2\beta\pm1$
			Obtain $2\sec^2\beta-9\sec\beta-5=0$	A1	condone absence of $=0$
			Attempt solution of 3-term quadratic in $\sec\beta$ to obtain at least one value of $\sec\beta$	M1	if factorising, factors must be such that expansion gives their first and third terms; if using formula, this must be correct for their values
			Obtain 5 with no errors in solution	A1	and, finally, no other value; no need to justify rejection of $-\frac{1}{2}$
				[4]	
			Or Attempt to express equation in terms of $\cos\beta$	M1	using identities which are correct apart maybe for sign slips
			Obtain $5\cos^2\beta+9\cos\beta-2=0$	A1	condone absence of $=0$
			Attempt solution of 3-term quadratic and show switch at least once to a secant value	M1	if factorising, factors must be such that expansion gives their first and third terms; if using formula, this must be correct for their values
			Obtain 5 with no errors in solution	A1 [4]	and, finally, no other value; no need to justify rejection of $-\frac{1}{2}$

Question			Answer	Marks	Guidance
3	(i)		Use α (possibly implicitly) to state that radius of 'base' is $\frac{1}{2}x$	*B1	or to obtain equiv such as $2r = x$ or $\frac{r}{x} = \frac{1}{2}$ or $\frac{x}{r} = 2$
			Substitute into formula to obtain $\frac{1}{3}\pi(\frac{1}{2}x)^2 x$ or $\frac{1}{3}\pi\frac{1}{4}x^2 x$ and obtain $\frac{1}{12}\pi x^3$	B1 [2]	dep *B; AG; necessary detail needed Note: comparing formulae $\frac{1}{3}\pi r^2 h$ and $\frac{1}{12}\pi x^3$ to 'deduce' is B0B0
3	(ii)		Differentiate to obtain $\frac{1}{4}\pi x^2$ or equiv	B1	whatever they call it
			Attempt division involving 14 and their value of derivative when $x = 8$	M1	ie $14 \div$ deriv or deriv $\div 14$ with $x = 8$
			Obtain 0.28	A1 [3]	allow 0.279 but not greater accuracy Alternatives: 1. $14t = \frac{1}{12}\pi x^3$ Obtain $\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{1}{56}\pi x^2$ <u>B1</u> Sub 8 and invert <u>M1</u> Ans <u>A1</u> 2. $x^3 = \frac{168t}{\pi}$ Obtain $3x^2\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{168}{\pi}$ <u>B1</u> Sub 8 <u>M1</u> Ans <u>A1</u>
4			Differentiate first term to obtain form $k(4x-7)^{-\frac{1}{2}}$	*M1	any non-zero constant k; M0 if this differentiation is carried out in the midst of some incorrect involved expression
			Obtain $2(4x-7)^{-\frac{1}{2}}$	A1	or (unsimplified) equiv
			Attempt use of quotient rule or, after adjustment, product rule	*M1	for QR, allow numerator wrong way round but needs $-$ sign in numerator; condone a single error such as absence of square in denominator, absence of brackets, …; for PR, condone no use of chain rule M0 if this differentiation is carried out in the midst of some incorrect involved expression
			Obtain $\frac{4(2x+1)-8x}{(2x+1)^2}$ or $4(2x+1)^{-1} - 8x(2x+1)^{-2}$	A1	or (unsimplified) equivs; give A0 if brackets absent unless subsequent calculation indicates their 'presence'
			Substitute 4 into expression for first derivative so that (initially at least) exactness is retained	M1	dep *M *M
			Obtain $\frac{58}{81}$	A1 [6]	answer must be exact Note: using $y = \sqrt{4x-7} + \frac{4}{2x+1}$: do not apply MR

Question			Answer	Marks	Guidance
5	(i)		Refer to translation and stretch	M1	in either order; ignore details here; allow any equiv wording (such as move or shift for translation) to describe geometrical transformation but not statements such as add 3 to x
			Either State translation in negative x-direction by 3	A1	or state translation by $\begin{pmatrix}-3\\0\end{pmatrix}$; accept horizontal to indicate direction; term 'translate' or 'translation' needed for award of A1
			State stretch by factor 2 in y-direction	A1	or parallel to y-axis or vertically; term 'stretch' needed for award of A1; these two transformations can be given in either order SC: if M0 but details of one transformation correct, award B1 for 1/3 (in Either, Or 1, Or 2 cases)
				[3]	
			Or 1 State stretch by factor $\frac{1}{2}$ in x-direction	A1	or parallel to x-axis; term 'stretch' needed for award of A1
			State translation in negative x-direction by 3	A1 [3]	or state translation by $\begin{pmatrix}-3\\0\end{pmatrix}$; term 'translate' or 'translation' needed for award of A1; these two transformations must be in this order – if details correct for M1A1A1 but order wrong, award M1A1A0
			Or 2 State translation in negative x-direction by 6	A1	or state translation by $\begin{pmatrix}-6\\0\end{pmatrix}$; term 'translate' or 'translation' needed for award of A1
			State stretch by factor $\frac{1}{2}$ in x-direction	A1 [3]	or parallel to x-axis; term 'stretch' needed for award of A1; these two transformations must be in this order – if details correct for M1A1A1 but order wrong, award M1A1A0
5	(ii)		Either Solve linear eqn/ineq to obtain critical value –6	B1	
			Attempt solution of linear eqn/ineq where signs of x and $2x$ are different	M1	
			Obtain critical value –2	A1	
			Attempt solution of inequality	M1	using table, sketch, …; implied by correct answer or answer of form $a < x < b$ or of form $x < a, x > b$ (where $a < b$); allow $\leq$ here
			Obtain $-6 < x < -2$	A1	as final answer; must be < not $\leq$; allow "$x > -6$ and $x < -2$"
				[5]	

Question			Answer	Marks	Guidance
			Or Square both sides to obtain $x^2 > 4(x^2 + 6x + 9)$	B1	or equiv
			Attempt solution of 3-term quadratic eqn/ineq	M1	with same guidelines as in Q2(ii) for factorising and formula
			Obtain critical values –6 and –2	A1	
			Attempt solution of inequality	M1	using table, sketch, …; implied by correct answer or answer of form $a < x < b$ or of form $x < a, x > b$ (where $a < b$); allow $\leq$ here
			Obtain $-6 < x < -2$	A1 [5]	as final answer; must be < not $\leq$; allow ' $x > -6$ and $x < -2$ '
6	**(i)**		Attempt evaluation involving *y* values	M1	with coefficients 1, 4 and 2 each occurring at least once; allow for wrong *y*-values; solution must include sufficient evidence of method
			Obtain $k(\ln 3 + 4\ln 7 + 2\ln 19 + 4\ln 39 + \ln 67)$	A1	any constant *k*; or decimal equivs; correct use of brackets required unless subsequent working shows their 'presence'
			Identify value of *k* as $\frac{2}{3}$	A1	as factor for their complete expression
			Obtain 22.4	A1 **[4]**	allow any value rounding to 22.4; answer only is 0/4
6	**(ii)**		State $9 + 6x^2 + x^4 = (3 + x^2)^2$	B1	or, if proceeding numerically, demonstrate in at least three cases that $\ln 9 = \ln 3^2, \ln 49 = \ln 7^2, \ln 361 = \ln 19^2, ...$
			Show relevant property $\ln(3 + x^2)^2 = 2\ln(3 + x^2)$ and conclude with value $2A$	B1 **[2]**	AG; necessary detail needed; if proceeding numerically, needs all five cases with relevant property Note: using Simpson's rule again here is B0B0
6	**(iii)**		Recognise $\ln(3e + ex^2)$ as $1 + \ln(3 + x^2)$	B1	
			Indicate in some way that $\int_0^8 1\,dx$ is 8 and conclude with value $A + 8$	B1 **[2]**	AG; necessary detail needed Note: using Simpson's rule again here is B0B0
7	**(i)**		State $y > 3$ or $f(x) > 3$ or $f > 3$ or 'greater than 3'	B1 **[1]**	must be > not $\geq$; allow $3 < y < \infty$

Question			Answer	Marks	Guidance
7	(ii)		Obtain expression or eqn involving $\ln(\frac{y-3}{4})$ or $\ln(\frac{x-3}{4})$	M1	or equivs such as $\ln(\frac{4}{y-3})$ or $\ln(\frac{4}{x-3})$
			Obtain $\ln(\frac{4}{x-3})$ or $-\ln(\frac{x-3}{4})$	A1	or equiv
			State domain is $x > 3$ or equiv	B1FT	following answer to part **(i) (**but with adjustment so that reference is to x)
			State range is all real numbers or equiv	B1 **[4]**	
7	(iii)		Obtain correct first iterate	B1	showing at least 3 dp; B0 if initial value not 3 but then M1A1A1 available
			Show correct iteration process	M1	showing at least 3 iterates in all; may be implied by plausible converging values; M1available if based on equation with just a slip in $x = \mathrm{f}(x)$ but M0 if based on clearly different equation
			Obtain at least 3 correct iterates	A1	allowing recovery after error; iterates to only 3 dp acceptable; values may be rounded or truncated
			Obtain (3.168, 3.168)	A1	each coordinate required to exactly 3 dp; award A0 if fewer than 4 iterates shown; part (iii) consisting of answer only gets 0 out of 4
			[3 → 3.199148.. → 3.163187.. → 3.169162.. → 3.168155.. → 3.168324..]	**[4]**	
7	(iv)		State P is point where the curves meet	B1 **[1]**	or equiv
8	(i)		Obtain $R = \sqrt{20}$ or $R = 4.47$	B1	
			Attempt to find value of α	M1	implied by correct value or its complement; allow sin/cos muddles; allow use of radians for M1; condone use of $\cos\alpha = 4, \sin\alpha = 2$ here but not for A1
			Obtain 26.6	A1 **[3]**	or greater accuracy 26.565…; with no wrong working seen
8	(ii)	(a)	Show correct process for finding one answer	M1	allowing for case where the answer is negative
			Obtain 21.3	A1FT	or greater accuracy 21.3045…; or anything rounding to 21.3 with no obvious error; following a wrong value of α but not wrong R
			Show correct process for finding second answer	M1	ie attempting fourth quadrant value minus α value
			Obtain 286 or 285.6	A1FT	or greater accuracy 285.5653…; or anything rounding to 286 with no obvious error; following a wrong value of α but not wrong R ;and no others between 0° and 360°
				[4]	

Question			Answer	Marks	Guidance
8	(ii)	(b)	State greatest value is 25	B1	allow if α incorrect
			Obtain value 63.4 clearly associated with correct greatest value	B1FT	or greater accuracy 63.4349…; following a wrong value of α
			State least value is 5	B1	allow if α incorrect
			Attempt to find θ from $\cos(\theta + \text{their}\alpha) = -1$	M1	and clearly associated with correct least value
			Obtain 153 or 153.4	A1FT	or greater accuracy 153.4349…; following a wrong value of α
				[5]	
9	(i)		Differentiate to obtain $2e^{2x} - 18$	B1	
			Equate first derivative to zero and use legitimate method to reach equation without e involved	M1	
			Confirm $x = \ln 3$	A1	AG; necessary detail needed (in particular, for solutions concluding $x = \frac{1}{2}\ln 9 = \ln 3$ or equiv award A0)
				[3]	
9	(ii)		Attempt integration	*M1	confirmed by at least one correct term
			Obtain $\frac{1}{2}e^{2x} - 9x^2 + 15x$	A1	or equiv
			Apply limits 0 and $\ln 3$ to obtain exact unsimplified expression	M1	dep *M
			Obtain $4 - 9(\ln 3)^2 + 15\ln 3$	A1	or exact (maybe unsimplified) equiv perhaps still involving e
			Attempt area of trapezium or equiv, retaining exactness throughout	M1	using $\frac{1}{2}\ln 3 \times (y_1 + y_2)$ where y_1 is 15 or 16 and y_2 is attempt at y-coordinate of Q; if using alternative approach involving rectangle and triangle, complete attempt needs to be seen for M1; another alternative approach involves equation of PQ ($y = \frac{8 - 18\ln 3}{\ln 3}x + 16$) with integration: M1 for attempting equation and integration, A1 for correct answer
			Obtain $\frac{1}{2}\ln 3 \times (16 + 24 - 18\ln 3)$	A1	or equiv perhaps still including e
			Subtract areas the right way round, retaining exactness	M1	dep on award of all three M marks
			Obtain $5\ln 3 - 4$	A1	or similarly simplified exact equiv
				[8]	